Plenary Conference

Graphes premiers : un tour d'horizon

Pierre.ILLE

CNRS Institut de Mathématiques de Marseille France

Abstract

Un graphe (simple) G est constitué d'un ensemble de sommets $S(G)$ et d'un ensemble d'arêtes $A(G)$, où une arête est une paire de sommets de G. La notion suivante de module apparaît naturellement lorsqu'on définit le quotient d'un graphe. Une partie M de $S(G)$ est un module de G lorsque pour tout $v \in S(G) \setminus M$, on a :

$$\{v, x\} \in A(G) \text{ pour tout } x \in M$$

ou

$$\{v, x\} \notin A(G) \text{ pour tout } x \in M.$$

Clairement, $\emptyset$, $\{u\}$ $(u \in S(G))$ et $S(G)$ sont des modules de G, appelés modules triviaux. Disons alors qu'un graphe est premier lorsque tous ses modules sont triviaux. Il est facile de voir qu'un graphe premier est connexe. L'inverse est faux lorsque le graphe connexe a trop d'arêtes.
Il n'est pas facile de décrire la structure d'un graphe premier car presque tous les graphes (finis) sont premiers. Néanmoins, on peut étudier les sous-graphes premiers dans un graphe premier. Par exemple, on peut s'intéresser à leurs tailles, à leur répartition, etc. Nous présentons les principaux résultats dans le cas fini, puis infini...

MODULES HOPFIENS OU COHOPFIENS HÉRÉDITAIRE

El Amin KAIDI

Université d' Almeria, Departement de Mathématiques
Almeria Spain

Abstract
Pour un anneau A, un A-module M est dit hopfien (respectivement cohopfien) héréditaire, si M, tous ses sousmodules et modules quotients sont hopfiens (respectivement cohopfiens). La classe des modules hopfiens (respectivement cohopfiens) héréditaires est strictement comprise entre la classe des modules noetheriens (respectivement artiniens) et la classe des modules hopfiens (repectivement cohopfiens). Plusieurrs théorèmes de structure de ces de modules, sur des anneaux particuliers, seront établits. Quelques questions ouvertes seront proposées.

Références

[1] A. Facchini, *Module Theory Endomorphism rings and direct sum decompositions in Some classes of modules*. Birkhäuser (1998).

[2] L. Fuchs Infinite *Abelian Groups Volume II*, Academic Press, Inc. (1973).

[3] A. Haily and A. Kaidi, *Modules with a ≪nice ≫ endomorphism ring and a new characterization of semisimple modules and rings*, Comm. In Algebra, 26(8), 2445-245 (1998).

[4] A. Haily and A. Kaidi. *Charactérisation de certaines classes d'anneaux par des propriétés des endomorphismes de leurs modules*, Comm. In Algebra, 27(10), 4943-495..(1999).

[5] A. Hmaimou, A. Kaidi and E. Sanchez, *Generalized Fitting modules and rings*, Journal of Algebra 308, 199-214 (2007)

[6] A. Kaidi and E. Sanchez *Strongly Fitting Modules and rings* Preprint.

[7] A. Kaidi et M. Sanghare *Une caractérisation des anneaux artiniens á idéaux principaux*, Lectures Notes in Mathematics, Springer-Verlag, 1328, 245-254 (1988)

[8] F. Kasch and A. Mader , Rings, Modules and the Total. Frontiers in Mathematics Birkhäuser (2004)

[9] P.A. Krylov and A. A. Tuganbaev , *Modules over Discrete Valuation Domains*. De Gruyter Expositions in Mathematics 43, Walter de Gruyter. Berlin. New York (2008)

Some properties of poly-Cauchy numbers with level 2

Takao Komatsu[1]

[1]Schooo of Science, Zhejiang Sci-Tech University, Hangzhou 310018 China

Poly-Cauchy numbers $\mathfrak{C}_n^{(k)}$ *with level* 2 are defined by

$$\mathrm{Lif}_{2,k}(\mathrm{arcsinh}t) = \sum_{n=0}^{\infty} \mathfrak{C}_n^{(k)} \frac{t^n}{n!}\,, \tag{1}$$

where arcsinh$t$ is the inverse hyperbolic sine function and

$$\mathrm{Lif}_{2,k}(z) = \sum_{m=0}^{\infty} \frac{z^{2m}}{(2m)!(2m+1)^k}\,.$$

This function is an analogue of *Polylogarithm factorial* or *Polyfactorial* function $\mathrm{Lif}_k(z)$, defined by

$$\mathrm{Lif}_k(z) = \sum_{m=0}^{\infty} \frac{z^m}{m!(m+1)^k}\,.$$

When $k = 1$ in (1), by $\mathrm{Lif}_{2,1}(z) = \sinh z/z$, the numbers $\mathfrak{C}_n^{(1)}$ are given by the generating function

$$\frac{t}{\mathrm{arcsinh}t} = \sum_{n=0}^{\infty} \mathfrak{C}_n^{(1)} \frac{t^n}{n!}\,. \tag{2}$$

Several initial values of $\mathfrak{C}_n^{(1)}$ are given as

$$\{\mathfrak{C}_{2n}^{(1)}\}_{n\ge 0} = 1, \frac{1}{3}, -\frac{17}{15}, \frac{367}{21}, -\frac{27859}{45}, \frac{1295803}{33}, -\frac{5329242827}{1365}, \dots .$$

Poly-Cauchy numbers with level 2 can be expressed in terms of multinomial coefficients with combinatorial summation, Stirling numbers of the first kind, or iterated integrals. We show several expressions, relations, and properties about poly-Cauchy numbers with level 2. When $k = 1$, we prove some more expressons in determinants, continued fractions or by Trudi's formula.

Références

[1] T. Komatsu, *Poly-Cauchy numbers*, Kyushu J. Math., Vol(67) : p143–p153.

[2] T. Komatsu and K. Kanamo, *Explicit formulae for sums of products of Bernoulli polynomials, including poly-Bernoulli polynomials*, Ramanujan J., Vol(33) : p301–p313.

[3] T. Komatsu and C. Pita-Ruiz, *Truncated Euler polynomials*, Math. Slovaca., Vol(68) : p527–p536.

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On amalgamated algebras along an ideal: A Survey

Najib Mahdou

Department of Mathematics, Faculty of Science and Technology of Fez,
Box 2202, University S. M. Ben Abdellah Fez, Morocco
mahdou@hotmail.com

Abstract. Let A and B be two rings with unity, let J be an ideal of B and let $f : A \to B$ be a ring homomorphism. In this setting, we can consider the following subring of $A \times B$:

$$A \bowtie^f J := \{(a, f(a) + j) \mid a \in A, j \in J\}$$

called the amalgamation of A with B along J with respect to f introduced by M. D'Anna, C. A. Finocchiaro and M. Fontana in 2009. This Talk is a survey about the amalgamation $A \bowtie^f J$.

References

[1] K. Alaoui Ismaili and N. Mahdou, Coherence in amalgamated algebra along an ideal, Bulletin of the Iranian Mathematical Society, Vol. 41 No. 3 (**2015**), 1–9.

[2] K. Alaoui Ismaili and N. Mahdou, On (n, d)- property in amalgamated algebra, Asian-European Journal of Mathematics, Vol. 9, No. 1 (2016).

[3] M. Chhiti and N. Mahdou, Some homological properties of amalgamated duplication of a ring along an ideal, Bulletin of the Iranian Mathematical Society, Vol. 38 No. 2 (2012) 507-515.

[4] M. Chhiti, N. Mahdou and M. Tamekkante, Self injective amalgamated duplication of a ring along an ideal, Journal of Algebra and Its Applications, Volume 12, Issue 07, November (2013), .

[5] M. Chhiti, M. Jarrar, S. Kabbaj and N. Mahdou, Prüfer conditions in an amalgamated duplication of a ring along an ideal, Comm. Algebra 43 (1) (2015) 249-261.

[6] M. D'Anna, C. A. Finocchiaro and M. Fontana, Amalgamated algebras along an ideal, in: Commutative Algebra and Applications, Proceedings of the Fifth International Fez Conference on Commutative Algebra and Applications, Fez, Morocco, 2008, W. de Gruyter Publisher, Berlin (2009), 155–172.

International Conference on Algebra and its Applications, (February 28-29, 2020), Faculty of Science Ain Chock, Casablanca

[7] M. D'Anna, C. Finocchiaro and M. Fontana, Properties of chains of prime ideals in an amalgamated algebra along an ideal, J. Pure Appl. Algebra 214 (9) (2010) 1633–1641.

[8] M. D'Anna, C. Finocchiaro and M. Fontana, New algebraic properties of an amalgamated algebra along an ideal, Comm. Algebra 44(5) (2016), 1836–1851.

[9] M. D'Anna, A construction of Gorenstein rings, J. Algebra 306(2) (2006), 507-519.

[10] M. D'Anna and M. Fontana, Amalgamated duplication of a ring along a multiplicative-canonical ideal, Ark. Mat. 45(2) (2007), 241-252.

[11] M. D'Anna and M. Fontana, An amalgamated duplication of a ring along an ideal: the basic properties, J. Algebra Appl. 6(3) (2007), 443-459.

[12] C. Finocchiaro, Prüfer-like conditions on an amalgamated algebra along an ideal, Houston J. Math. 40(1) (2014), 63–79

[13] C. Finocchiaro, Amalgamation of algebras and the ultrafilter topology on the space of valuation overrings of an integral domain, PhD thesis, University Roma Tre, Rome, December 2010.

[14] M. Kabbour and N. Mahdou, Amalgamation of rings defined by Bézout-like conditions, Journal of Algebra and its Applications, Vol. 10 (2011) 1343 - 1350.

[15] N. Mahdou, M. Tamekkante and S. Yassemi, Coherent power series ring and weak Gorenstein global dimension, Glasgow Mathematical Journal, Vol. 55, (2013), 533-536.

[16] N. Mahdou, A. Mimouni and M. A. Moutui, On almost valuation and almost Bézout rings, Comm. Algebra, 43 (2015) No. 1, 297-308.

[17] N. Mahdou and M. Tamekkante, Gorenstein global dimension of an amalgamated duplication of a coherent ring along an ideal, Mediterranean Journal of Mathematics (Springer), Vol. 8 (2011) 293-305.

[18] N. Mahdou and M. A. Moutui, fqp-property in amalgamated algebras along an ideal, Asian-European Journal of Mathematics, Vol. 8, No. 3 (2015).

[19] H. Maimani and S. Yassemi, Zero-divisor graphs of amalgamated duplication of a ring along an ideal, J. Pure Appl. Algebra 212 (1) (2008) 168174.

Pattern recognition via Artin transfers, applied to p-class field towers

Daniel C. Mayer[1]

[1]Laboratoire de Mathématique, Université Charles François, Graz, Austria

Résumé/Abstract

The *strategy of pattern recognition* by means of kernels and targets of Artin transfers was founded by myself in 2009 and developed systematically in the past ten years. It is a progressive technique for determining the structure of the various stages, $\mathrm{Gal}(F^{(n)}/F)$, $n \geq 1$, of the p-class tower, $F = F^{(0)} \leq F^{(1)} \leq F^{(2)} \leq \ldots \leq F^{(n)} \leq \ldots$, of an algebraic number field $F/\mathbb{Q}$ for a prime number p. Whereas for $n \geq 3$ non-abelian *iterated Artin patterns of higher order* with increasing complexity are required, it suffices to know the abelian *Artin pattern of first order*, $\mathrm{AP}(G) = (\kappa(G), \tau(G))$, for the identification of the metabelianization, that is the second derived quotient, $M = \mathrm{Gal}(F^{(2)}/F) \simeq G/G''$, of the full tower group $G = \mathrm{Gal}(F^{(\infty)}/F)$ of the maximal unramified pro-p extension $F^{(\infty)} = \cup_{n\geq 1} F^{(n)}$ of F. According to the Artin reciprocity law, the latter can be computed numerically with the aid of kernels $\kappa(G) = (\ker(T_{F,E}))_E$ and targets $\tau(G) = (\mathrm{Cl}_p(E))_E$ of extension homomorphisms $T_{F,E} : \mathrm{Cl}_p(F) \to \mathrm{Cl}_p(E)$ of p-classes from F into all abelian unramified p-extensions $F \leq E \leq F^{(1)}$. The strategy has proved to be an outstanding innovation in computational class field theory and has been applied by myself and my international collaborators to base fields F with numerous types of p-class groups $\mathrm{Cl}_p(F)$ and primes $p \in \{2, 3, 5, 7\}$, starting with $(3, 3)$ in 2009 [1] and $(9, 3)$ in 2011, extended to three stages with Boston, Bush in 2012 [2], over $(2, 2, 2)$ with Azizi, Zekhnini, Taous in 2014 [3] and $(5, 5)$ with Azizi, Kishi, Talbi, Talbi in 2015 [4], up to the multi-layered situations $(4, 4)$ with Newman and $(9, 9)$, $(27, 3)$, $(81, 3)$ by myself in 2019, which led to my discovery of the surprising phenomenon of *harmonically balanced capitulation kernels*.
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Références

[1] D. C. Mayer, *Principalization algorithm via class group structure*, J. Théor. Nombres Bordeaux, Vol. 26 (2014) : 415–464.

[2] M. R. Bush and D. C. Mayer, 3-*class field towers of exact length* 3, J. Number Theory, Vol. 147 (2015) : 766–777.

[3] A. Azizi, A. Zekhnini, M. Taous and D. C. Mayer, *Principalization of* 2-*class groups of type* $(2, 2, 2)$ *of biquadratic fields* $\mathbb{Q}(\sqrt{p_1 p_2 q}, \sqrt{-1})$, Int. J. Number Theory, Vol. 11 (2015) : 1177–1215.

[4] A. Azizi, Y. Kishi, D. C. Mayer, M. Talbi and Mm. Talbi, 5-*Class towers of cyclic quartic fields arising from quintic reflection*, Ann. Math. Québec (2019), DOI :10.1007/s40316-019-00125-2.

Cyber Security and Cryptography

Abderrahmane NITAJ

University of Caen, France

Abstract
Cyber security is essential for protecting sensitive information systems and platforms shared by a large number of users against external attacks. The implementation of cyber security systems depends on several factors such as the use of solid software and hardware products. Cryptography plays an important role in cyber security to protect information systems and to guarantee the good functioning of several applications such as Internet voting, medical records, e-commerce and e-money services. Cryptography is the technique of transforming and storing or transmitting confidential data in an enciphered way so that only authorized users can decipher and retrieve it as in the original form. In this talk, we will discuss the contribution of cryptography in cyber security for the protection of communications, information systems and e-commerce.