Session : Algebra And Algebraic Topology

k-Symplectic Affine Lie Algebras

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Abstract

The notion of k-symplectic structures was introduced by A. Awane in his dissertation in 1984 ([1],[2]). Here we are interested by the classification of Lie algebras provided with such a structure. We introduce also the notion of affine structure associated to a k-symplectic structure on a Lie algebra.

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Capacities in fractional Sobolev spaces with variable exponents

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Abstract

The concept of capacity is indispensable to an understanding point-wise behavior of functions in a Sobolev space. In a sense, capacity is a measure of size for sets and they measure small sets more precisely than the usual Lebesgue measure.

Sobolev spaces and capacities theory is one of the significant aspects of fine topology, and the classical and fine nonlinear potential theory. In this setting, there are two natural kinds of capacities : Sobolev capacity and relative capacity. Both capacities have their advantages.

In this paper we develop a capacities theory connected with the fractional Sobolev spaces with variable exponents. Fundamental proprieties of capacity including Choquet capacity, capacitability and several results, are studied.

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Semiderivations and generalized semiderivations in near-rings

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Abstract

Let N be a zero-symmetric prime near-ring. An additive mapping $F : N \to N$ is said to be a generalized semiderivation associated with a semiderivation d and a map g if it satisfies $F(xy) = F(x)y + g(x)d(y) = d(x)g(y) + xF(y)$ and $F(g(x)) = g(F(x))$ for all $x, y \in N$. The purpose of this paper is to extend some results concerning generalized derivations of prime near rings to generalized semiderivations. Moreover, example is provided to show the necessity for N to be prime and g to be an automorphism in the hypothesis of the theorems. When, $g = id_N$, one can easily obtain the main results of [5] and [9].

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Relatively Cyclic *P*-Contractions in Locally K-Convex Space

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Abstract :

Our main goal of this research is to present the theory of points for relatively cyclic and relatively noncyclic p-contractions in complete locally K{convex spaces by providing basic conditions to ensure the existence and uniqueness of _xed points and best proximity points of the relatively cyclic and relatively noncyclic p-contractions map in locally K-convex space.

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Fixed Point Theorems of Block Operator Matrices On Banach Algebras

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Abstract

Fixed point theory is one of the famous and traditional theories in mathematics and has a large number of applications in various fields of pure and applied mathematics, as well as in physical, chemical, life and social sciences. In this work we are concerned with fixed point results on Banach algebras of operators defined by a 2×2 block operator matrix

$$\begin{pmatrix} A & B.B' \\ C & D \end{pmatrix}$$

where the entries of the matrix are in general nonlinear operators defined on Banach algebras. Our results are formulated using the weak topology.

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Classification of links up to link-homotpy using Milnor invariants

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Abstract

One of the main problems of Knot theory is the classification of links (i.e. embeddings of several disjoint circles into the space,). Links are classified up to many equivalence relations. Link-homotopy, which was introduced by Milnor in 1954 [1], is one of these equivalence relations. Two links are link-homotopy equivalent if one can be transformed into the other by a finite sequence of ambient isotopies where no crossing change is allowed between distinct components of the link but crossing changes are allowed on the same component. In [1] Milnor classified links with up to three components using some numerical invariants called *Milnor Invariants.*. A complete classification of links up to link homotopy was given in [2] by Habegger and Lin in 1990. In this talk we will introduce Milnor's definition of these invariants and give his classification of links with up to three components up to link-homotopy.

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ON THE TORAL RANK CONJECTURE AND SOME CONSEQUENCES

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Abstract

The aim of this work is to improve the lower bound of the Puppe inequality. His theorem [15, Theorem 1.1] states that the sum of all Betti numbers of a well-behaved space X is at least equal to 2n, where n is rank of an n-torus T^n acting almost freely on X.

Key words: Puppe inequality, Betti numbers, almost free action of a maximal torus, rational homotopy groups, rational cohomology groups.
AMS subject classification: 55P62

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P-Coloring of Knots and Links

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Abstract

One of the main problems in knot theory is determining whether two knots are equivalent or not Thus, knot invariants are constructed to distinguish between different knots. One of these invariants is called p-colorability of knots. In 1961 Fox [1] introduced a method of coloring diagrams of knots by Zp (the integers modulo p). In 1990, Harary and Kauffman [2] defined the minimum number of colors of a p-colorable knot with considering p as an odd prime, this minimum number is also a knot invariant and it is in general hard to calculate, for this reason Takuji Nakamura, Yasutaka Nakanishi, and Shin Satoh introduced [3] the notion of a graph associated with a Fox p-coloring of a knot to show that any non-trivial p-coloring requires at least $\lfloor log_2 p \rfloor + 2$ colors. In this talk, we will introduce the definition of the minimum number of colors needed to produce a non-trivial p-coloring of a knot and the lower bound to estimate this minimum number.

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A constructive proof of Khamsi-Kirk-Pouzet fixed point theorem

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Abstract

In this work, we prove that if a generalized metric space (E, d) has a compact and normal structure then every nonexpansive mapping has a fixed point. Our proof differs from that given by the authors in [1], since it adapt a constructive lemma due to Gillespie and Williams [2]. As usual, we obtain Tarski's fixed point theorem as a corollary.

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Homoderivations and Jordan right ideals in 3-prime near-rings

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Abstract

Let $\mathcal{N}$ be a prime near-ring with center $Z(\mathcal{N})$ and $\mathcal{J}$ a nonzero Jordan ideal of $\mathcal{N}$. The aim of this paper is to prove some theorems showing that $\mathcal{N}$ must be commutative if it admits a homoderivation h satisfying any one of the following properties : $(i)h(\mathcal{N}) \subseteq Z(\mathcal{N}), (ii)h([x, y] = 0, (iii)h([x, y] = [x, y]$ for all $x, y \in N$ and $(iv)h(\mathcal{J}) = \{0\}$.
On the other hand, we show that there is no nonzero homoderivation h satisfying any one of the following properties : $(v)h(x \circ y) = 0, (vi)h(x \circ y) = x \circ y, (vii)h([x, y]) = x \circ y$ for all $x, y \in N$, and $(viii)h(j) = j, (ix)h(ij) = ij$ for all $j \in \mathcal{J}$.
Moreover, we give some examples which show that the hypotheses placed in our results are not superfluous.

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