Session : Algebra And Graph Theory

A NOTE ON w-SPLIT MODULES

REFAT ABDELMAWLA KHALED ASSAAD[1] and FUAD ALI AHMED ALMAHDI[2]

[1]Department of Mathematics, Faculty of Science, University Moulay Ismail Meknes, Morocco
[2]Department of Mathematics, Faculty of Science, King Khalid University Abha, Saudi Arabia

Abstract

In this communication, we introduce and study some properties of w-split modules. And we use these modules to characterize some classical rings, for example we will prove that a ring R is von Neumann regular rings if and only if every finitely presented R-module is w-split, and R is semi-simple ring if and only if every R-module is w-split. And we introduce the w-split dimensions of modules. The relation between projective dimension and w-split dimension will be discussed.

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SOME FACTORIZATION PROPERTIES OF THE COMPOSITE RING $A + B[\varGamma^*]$

B. Boulayat[1], S. El Baghdadi[1]

[1]Laboratoire de Mathématiques et Applications, FST, Beni Mellal

Résumé

Let $A \subseteq B$ denote an extension of commutative integral rings with identity, $\varGamma$ a nonzero torsion-free (additive) grading monoid with $\varGamma \cap -\varGamma = \{0\}$ and $\varGamma^* = \varGamma \setminus \{0\}$. $B[\Gamma]$ is the semigroup ring of Γ over B. In this talk, we will discuss some factorization properties of the pullback $A+B[\Gamma^*] = \{f \in B[\Gamma] | f(0) \in A\}$.

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STRONGLY PRIMARY IDEALS IN RINGS WITH ZERO-DIVISORS

A. Elkhalfi, N. Mahdou and Y. Zahir

Laboratory of Modeling and Mathematical Structures,
Department of Mathematics, Faculty of Science and Technology of Fez,
Box 2202, University S.M. Ben Abdellah Fez, Morocco.

Résumé/Abstract

Let A be an integral domain with quotient field K. Badawi and Houston called a strongly primary ideal I of A if whenever $x, y \in K$ and $xy \in I$, we have $x \in I$ or $y^n \in I$ for some $n \geq 1$. In this note, we study the generalization of strongly primary ideal to the context of arbitrary commutative rings. We define a primary ideal of A to be strongly primary if for each $a, b \in A$, we have $aP \subseteq bA$ or $b^n A \subseteq a^n P$ for some $n \geq 1$.

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On Armendariz-like properties

L. ES-SALHI[1] and M. CHHITI[2]

[1]Laboratoire "Modélisation et Structures Mathématiques", Université Sidi mohamed ben abdellah, Fes
[2]Laboratoire "Modélisation et Structures Mathématiques", Université Sidi mohamed ben abdellah, Fes

Résumé/Abstract

Let $f : A \to B$ be a commutative ring homomorphism and let J be an ideal of B .
The amalgamation of A with B along J with respect to f is the subring of $A \times B$ given by

$$A \bowtie^f J := \{(a, f(a) + j) | a \in A, j \in J\}$$

In this talk , we give a characterization for $A \bowtie^f J$ to be an Armendariz ring, nil Armendariz ring and weak ring.

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Residual coordinate over one-dimensional rings

M. El Kahoui, N. Essamaoui and M. Ouali

Departement of Mathematics, Faculty of Sciences Semlalia
Cadi Ayyad university, Marrakesh.

Abstract

This is a joint work with my PhD project supervisors M'hammed El Kahoui and Mustapha Ouali.

Throughout, all considered rings are commutative with unity. Given a ring R we denote by $R^{[n]}$ the polynomial ring in n variables over R. A polynomial q in $A = R^{[n]}$ is said to be a coordinate if $A = R[q]^{[n-1]}$, i.e., q is a component of an R-automorphism of $R^{[n]}$. It is said to be a stable coordinate if q is a coordinate of $R^{[n+m]}$ for some $m \geq 1$.

Given a prime ideal $\mathfrak{p}$ of R, we denote by $K(\mathfrak{p})$ the residue field $R_{\mathfrak{p}}/\mathfrak{p}R_{\mathfrak{p}}$. A polynomial q in A is said to be a residual coordinate if for every prime ideal $\mathfrak{p}$ of R we have $K(\mathfrak{p}) \otimes_R A = (K(\mathfrak{p}) \otimes_R R[q])^{[n-1]}$. A natural question that stands for a long time is whether a given polynomial q in $R^{[n]}$ is a coordinate or at least a stable coordinate. For $n = 2$, Bhatwadekar and Dutta [1] proved that every residual coordinate is a stable coordinate over any noetherian ring. In a work of van Rossum and van den Essen [5] the noetherianity assumption is dropped when R contains $\mathbb{Q}$. In 2014, this result has been generalized by Das and Dutta [2] to higher dimension $n \geq 3$. But the question of how many variables need to be added in not treated neither in [1] nor in [2]. In the particular case $R = K^{[1]}$, where K is algebraically closed and contains the rationals, El Kahoui and Ouali [4] proved that every residual coordinate over R is a 1-stable coordinate. More recently, Dutta and Lahiri [3] showed that the same result holds if R is an affine algebra, over an algebraically closed field K, of Krull dimension 1 such that either K contains $\mathbb{Q}$ or R_{red} is seminormal. In this talk we will show that such a result still holds over a large class of one-dimensional rings including arbitrary affine algebras over algebraically closed fields as well as noetherian complete local rings containing a field.

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On the spectral reconstruction problem for digraphs

E. Bankoussou-mabiala[1], A. Boussaïri[1], A. Chaïchaâ[1], B. Chergui[1] and S. Lakhlifi[1]

[1]Laboratoire de Topologie, Algèbre, Géométrie et Mathématiques Discrètes, Faculté des Sciences Aïn Chock, Université Hassan II, Casablanca

Résumé/Abstract

The idiosyncratic polynomial of a graph G with adjacency matrix A is the characteristic polynomial of the matrix $A + y(J - A - I)$, where I is the identity matrix and J is the all-ones matrix. It follows from a theorem of Hagos (2000) combined with an earlier result of Johnson and Newman (1980) that the idiosyncratic polynomial of a graph is reconstructible from the multiset of the idiosyncratic polynomial of its vertex-deleted subgraphs.For a digraph G with adjacency matrix A, we define its idiosyncratic polynomial as the characteristic polynomial of the matrix $A+y(J-A-I)+zA^t$. By forbidding two fixed digraphs on three vertices as induced subdigraphs, we prove that the idiosyncratic polynomial of a digraph is reconstructible from the multiset of the idiosyncratic polynomial of its induced subdigraphs on three vertices. As an immediate consequence, the idiosyncratic polynomial of a tournament is reconstructible from the collection of its 3-cycles. Another consequence is that all the transitive orientations of a comparability graph have the same idiosyncratic polynomial.

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The $\mathcal{F} - \lim$ of a collection of zero dimensional rings

Hassan Mouadi[1], Driss Karim[1]

[1]Faculty of Sciences and Technology
University Hassan 2 of Casablanca
19, Tarik Bnou Ziad Street, Hospitals, Casablanca Morocco

Abstract

Let R be a subring of a ring S. The first goal of this work is to define the $\mathcal{F} - \lim$ of the set $\mathcal{Z}(R, S)$. Then we give a characterization of $\mathcal{Z}(R, S)$ by using the $\mathcal{F} - \lim$. Thereby, its relationship with ultrafilter limit and the direct limit of a family of rings.

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SUR LE GRAPHE DE PALEY

ZHOUR OUMAZOUZ, DRISS KARIM,

[1]Laboratoire De Mathématiques et applications, Université Hassan 2, FSTM Mohammedia

Résumé /Abstract :

Parmi les protocoles de la cryptographie quantique existants, on compte notamment les protocoles de partage de secret. Ils consistent en la distribution d'un secret classique ou quantique entre plusieurs personnes qui doivent se concerter pour pouvoir y accéder. Les protocoles de partage de secret constituent une primitive cryptographique dont le cas classique a été largement traité mais dont les analogues quantiques laissent la place à plusieurs améliorations.

Notre travail traite de l'utilisation des états graphes dans les protocoles de partage de secret quantique et de l'étude des structures des graphes associés.

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Graded almost pseudo-valuation domains

C. Bakkari[1], N. Mahdou[2] and A. Riffi[1,2]

[1]Faculty of Science, University Moulay Ismail, Meknes
[2]Faculty of Science and Technology, University S. M. Ben Abdellah, Fez.

Résumé/Abstract

let Γ be an arbitrary torsionless grading monoid. In this talk, we introduce the notion of Γ-graded almost pseudo-valuation domains, and we provide their elementary properties.

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On (-2)-spectrally monomorphic tournaments related to their adjacency matrices

Abderrahim BOUSSAIRI[1], Abdelhak CHAICHAA[2] and Imane SOUKTANI[1,2]

[1]Laboratoire TAGMED, UniversitÃ© Hassan II, Casablanca
[2]Laboratoire TAGMED, UniversitÃ© Hassan II, Casablanca

Abstract

Let $T = (V, A)$ be a tournament on n vertices. The adjacency matrix of the tournament T is the $n \times n$ matrix $A = (a_{ij})$ in which $a_{ij} = 1$ if $(v_i, v_j) \in A$ and 0 otherwhise. A tournament matrix is a $\{0,1\}$-matrix A such that $A + A^t = J - I$, where I and J will (respectively) denote the $n \times n$ identity matrix and all-ones matrix. We say that a tournament matrix is (-2)-monomorphic if all its principal submatrices of order $n-2$ are isomorphic. Moreover, we say that it is (-2)-spectrally monomorphic if all its principal submatrices of order $n-2$ have the same characteristic polynomials. In the present paper, we give a characterization of (-2)-spectrally monomorphic tournaments related to their adjacency matrices.

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extensions of graded rings

M. E. Charkani [1], [2] and A.soullami [1,2]

[1]Department of Mathematics, Faculty of Sciences, Dhar-Mahraz, Fes, Morocco.
[2]Department of Mathematics, Faculty of Sciences, Dhar-Mahraz, Fes, Morocco.

Résumé/Abstract

Throughout this work R be a commutative ring and Γ a semigroup which acts on the multiplicative semigroup $(R^*, \times)$.For any 2-cocycle α in $Z^2(\Gamma, R^*)$, we define the weak crossed product $A = R[\Gamma, \alpha]$ as the Γ-graded R-algebra $A = \oplus_{\sigma\in\Gamma} A_\sigma = \oplus_{\sigma\in\Gamma} Ru_\sigma$ where $u_\sigma u_\tau = \alpha(\sigma, \tau)u_{\sigma\tau}$ and $u_\sigma r = r^\sigma u_\sigma$. Our main interest will be to study some Galois extensions of the weak crossed product $A = R[\Gamma, \alpha]$. Such study required to characterize the separability of graded rings extensions, graded projective $R[\Gamma, \alpha]$-modules and to search the structure and generators of fixed rings. In this work we generalize some new results concerning Galois extensions over graded rings

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On SA-rings

N. Mahdou[1] and Y. Zahir[1]

[1] Laboratory of Modeling and Mathematical Structures, Sidi Mohamed Ben Abdellah University, Fez

Abstract

Let R be a commutative ring with identity. In this paper, we pursue the investigation of the SA-ring property introduced in [2], in the context of commutative ring with identity. A ring A is called SA-ring if for every two ideals I, J of R there is an ideal K such that $Ann(I) + Ann(J) = Ann(K)$. Specificaly, our field of intrest is to examine how this property behave with respect to localization, direct product, trivial ring extension and the amalgamation of rings along an ideal. Along the way, we give new and original examples of SA-rings.

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About the realizability of 3-uniform hypergraphs

Abderrahim Boussaïri[a 1] Brahim Chergui[a 2] Pierre Ille[b 3] Mohamed Zaidi[a 4]

[a]Faculté des Sciences Aïn Chock, Département de Mathématiques et Informatique,
Km 8 route d'El Jadida, BP 5366 Maarif, Casablanca, Maroc
[b]Aix Marseille Univ, CNRS, Centrale Marseille, I2M, Marseille, France

Abstract

Let H be a 3-uniform hypergraph. A tournament T defined on $V(T) = V(H)$ is a realization of H if the edges of H are exactly the 3-element subsets of $V(T)$ that induce 3-cycles. We characterize the 3-uniform hypergraphs that admit realizations by using a suitable modular decomposition.

Key words : hypergraph, 3-uniform, module, tournament, realization..
AMS subject classification : (2010) : 05C65, 05C20.

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1. E-mail : aboussairi@hotmail.com
2. E-mail : cherguibrahim@gmail.com
3. E-mail : pierre.ille@univ-amu.fr
4. E-mail : zaidi.fsac@gmail.com