Session : Algebra And Harmonic Analysis

Some important results concerning quadratic type functional equations on semigroups

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Abstract

Let $(S,+)$ be an abelian semigroup, let σ be an involution of S, let X be a linear space over the field $\mathbb{K}\in\{\mathbb{R},\mathbb{C}\}$ and let μ,ν be linear combination of Dirac measures. In the present paper, we find the general solution $f,g: S\to X$ of the following functional equation

$$\int_S f(x+y+t)d\mu(t)+\int_S f(x+\sigma(y)+t)d\nu(t)=f(x)+g(y),\quad x,y\in S,$$

in terms of additive and bi-additive maps. Many consequences of this result are presented.

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ON CLASSES OF HARMONIC FUNCTIONS OF CARLEMAN TYPE

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Abstract

Let f be harmonic functions on the unit disk $\mathbb{D}$, of the complex plane $\mathbb{C}$. We show that f can be expanded in a series $f = \sum_n f_n$, where f_n is a harmonic function on $\mathbb{D}_{n,\Gamma,A}$ satisfying $\sup_{z\in\mathbb{D}_{n,\Gamma,A}} \mid f_n(z) \mid\leq C\rho^n$ for some constants $C > 0$ and $0 < \rho < 1$, and where $(\mathbb{D}_{n,\Gamma,A})_n$ is a suitably chosen sequence of decreasing neighborhoods of the closure of $\mathbb{D}$. Conversely, if f admits such an expansion then f is of Carleman type. The decrease of the sequence $(\mathbb{D}_{n,\Gamma,A})_n$ characterizes the smoothness of f. These constructions are perfectly explicit.

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Some Results of $*$-K-g frames in Hilbert C^*-module

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RÃ©sumÃ©/Abstract

Frame theory is recently an active research area in mathematics, computer science, and engineering with many exciting applications in a variety of different fields. Frames were first introduced in 1952 by Duffin and Schaefer in the study of nonharmonic fourier series. The theory of frames has been generalized rapidly and various generalizations of frames in Hilbert spaces and Hilbert C^*-modules.
In this research work, we study some properties of the $*$-K-g-frame in Hilbert C^*-modules and we establish some new results.

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On linear Dynamical Systems of elementary operators

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Abstract

Let X be a Banach space with $\dim X > 1$ such that its topological dual X^* is separable and $\mathcal{B}(X)$ the algebra of all bounded linear operators on X. In the present work, we introduce the notion of mixing recurrent and we investigate the study of recurrent and mixing recurrent for elementary operators on an admissible Banach ideal $(J, \|.\|_J)$ of $\mathcal{B}(X)$. Also, we study the passage of property of being supercyclic from an operator $T \in \mathcal{B}(X)$ to the left and the right multiplication L_T and R_T induced by T on an admissible Banach ideal of operators $(J, \|.\|_J)$. In particular, we show that

(i) T satisfies the supercyclicity criterion on X if and only if L_T is supercyclic on $(J, \|.\|_J)$.

(ii) T^* satisfies the supercyclicity criterion on X^* if and only if R_T is supercyclic on $(J, \|.\|_J)$.

(iii) $T \oplus T$ is recurrent on $X \bigoplus X$ if and only if L_T is recurrent on $(J, \|.\|_J)$.

(iv) T is mixing recurrent on X if and only if L_T is mixing recurrent on $(J, \|.\|_J)$.

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Continuous Frame With C^*-Valued Bounds

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Abstract

Frame theory is an exciting, dynamic and fast paced subject with applications in numerous
elds of mathematics and engineering.
In this talk, we study Continuous Frame and introduce Continuous Frame with C^*-valued bounds.
Also, we establich some properties.

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Degenerate parabolic problems with variable exponent and L^1-data

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Abstract

Let $\Omega \subset \mathbb{R}^d, (d \geq 2)$ be a open bounded domain with a connected Lipschitz boundary $\partial\Omega$ and T be a fixed positive real number. Our aim of this communication is to prove existence results of entropy solutions for the nonlinear degenerate parabolic problem with variable exponent

$$\begin{cases} \dfrac{\partial u}{\partial t} - \operatorname{div}\left(\omega|\nabla u|^{p(.)-2}\nabla u\right) = f \quad in \quad Q_T :=]0,\ T[\times\Omega, \\ u = 0 \quad on \quad \Sigma_T :=]0,\ T[\times\partial\Omega, \\ u(.,0) = u_0 \quad in \quad \Omega, \end{cases}$$

where $p(.)$ is a continuous function defined on $\overline{\Omega}$ with $p(x) > 1$ for all $x \in \overline{\Omega}$ and ω is a measurable function on Ω, strictly positive and satisfying the following hypotheses

(H_1) : $\omega \in L^1_{loc}(\Omega)$ and $\omega^{\frac{-1}{p(x)-1}} \in L^1_{loc}(\Omega)$,

(H_2) : $\omega^{-s(x)} \in L^1_{loc}(\Omega)$ where $s(x) \in \left(\frac{N}{p(x)}, \infty\right) \cap \left(\frac{1}{p(x)-1}, \infty\right)$.

The datum f is in $L^1(\Omega)$.

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Les applications surjectives préservant le sous espace spectral local pour la somme, le produit, le produit triple et le produit généralisé

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Abstract
In this talk we discuss the form of surjective maps from $\mathcal{B}(X)$ into itself satisfying

$$X_{\phi(A)*\phi(B)}(\{\lambda\}) = X_{A*B}(\{\lambda\})$$

for all $A, B \in \mathcal{B}(X)$ and all $\lambda \in \mathbb{C}$, where $X_A(\{\lambda\})$ is the local spectral subspace of A associated with $\{\lambda\}$ and $A * B$ is one of the for different kinds of binary operations on operators : The sum $A + B$, the product AB, triple product ABA and generalized product of operators.

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Sur les séries de Dirichlet multiples

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Résumé
Soit $f_r : \mathbb{N}^r \longrightarrow \mathbb{C}$ une fonction arithmétique de r variables, où $r \geq 2$. Nous étudions les séries de Dirichlet multiples définies par

$$D(f_r, s_1, \cdots, s_r) = \sum_{\substack{n_1,\cdots,n_r=1 \\ (n_1,\cdots,n_r)=1}}^{\infty} \frac{f_r(n_1, \cdots, n_r)}{n_1^{s_1} \cdots n_r^{s_r}}.$$

où $f_r(n_1, \cdots, n_r) = f(n_1)f(n_2) \cdots f(n_r)$ et f est une fonction arithmétique complètement multiplicative ou spécialement multiplicative d'une seule variable. Nous obtenons des formules pour ces séries exprimées par un produit fini sur tous les nombres premiers et les fonctions L de Dirichlet. La preuve utilise la formule du produit eulérien généralisé. De plus, nous appliquons ces formules sur quelques séries de Dirichlet multiples associées à certaines fonctions complètement multiplicatives et spécialement multiplicatives, et exprimons ces séries par la fonction zêta de Riemann.

Abstract
Let $f_r : \mathbb{N}^r \longrightarrow \mathbb{C}$ be an arithmetic function of r variables, where $r \geq 2$. We study the multiple Dirichlet series defined by

$$D(f_r, s_1, \cdots, s_r) = \sum_{\substack{n_1,\cdots,n_r=1 \\ (n_1,\cdots,n_r)=1}}^{\infty} \frac{f_r(n_1, \cdots, n_r)}{n_1^{s_1} \cdots n_r^{s_r}}.$$

where $f_r(n_1, \cdots, n_r) = f(n_1)f(n_2) \cdots f(n_r)$ and f is a completely or specially arithmetic function of a single variable. We obtain formulas for these series expressed by an infinite product over all prime numbers and the Dirichlet L-functions. The proof use the formula of Eulerian product generalized. In addition, we apply these formulas on the multiple Dirichlet series associeted of certain completely multiplicative functions and specially multiplicative functions, and express these series by the Riemann zeta function.

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Representation theorem of set valued regular martingale and application

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In this communication, we present a new theorem concerning a representation of set valued regular martingales, the proof is based on martingale selectors approach. As applications, various convergence results of set valued martingales are provided.
The notion of multivalued martingale is an extension of real and vector martingales; In fact, the values of random variables involved are closed and convex subsets of a normed space, instead of real numbers or vectors. Multivalued martingale has been studied by many authors; see Akhiat et al, (2010); Choukairi,(1990); Hess, (1991); Neveu, (1972); Tahri, (2012); Wang and Xue, (1994) and so on. In particular, Hiai and Umegaki, (1977) presented the theory of set valued conditional expectations which was one of the basic foundation of the study of set valued martingale. It is well known in the literature that uniformly intergrable martingale with values in RNP Banach space is a regular martingale. See Egghe, (1984); Chatterji, (1968); Neveu, (1972). This result has been extended by Hiai and Umegaki, (1977) to bounded set valued martingale with values in convex and closed subset of RNP Banach space E with strongly separable dual E^*. Farther Choukairi, (1990) has studied the same problem where E is reflexive and separable. The purpose of this work is to go on with the study of multivalued martingales (particulary, regular martingales) with closed convex values in a separable Banach space E.

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Controlled continuous g-frames in Hilbert C^*-module

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Abstract

Frame Theory has a great revolution in recent years, this Theory have been extended from Hilbert spaces to Hilbert C^*-modules. We study of the concept of Controlled Continuous g-Frames in Hilbert C^*-Modules. Also we give some properties.

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