

## **Session : Algebra and Computer Security**

## Decoding LDPC Codes by Information Geometry and Alternating Projections

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### Abstract

We present an algorithm for decoding LDPC codes based on information geometry as introduced by Ikeda et al. For the purpose of the implementation of this algorithm, we explicitly compute the m-projection to an e-flat submanifold, in the case of a binary symmetric channel and the Gaussian channel. The same paradigm is applicable to Turbo decoding, which is a manifestation of the belief propagation algorithm. We also give a theorem based on alternating projections in the realm of information geometry, which is compared to alternating projections in the realm of convex programming.

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## Half-integral weight modular forms and real quadratic $p$ -rational fields

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### Résumé/Abstract

Using half-integral weight modular forms we give a criterion for the existence of real quadratic  $p$ -rational fields [2, 3]. Modular forms are particular holomorphic functions whose Fourier coefficients are most of time interesting arithmetic functions. Here we use Cohen-Eisenstein series which gives special values of  $L$ -functions of quadratic fields as Fourier coefficients [1]. We give our results after studying the  $p$ -adic valuation of these coefficients. For  $p = 5$  we prove the existence of infinitely many real quadratic  $p$ -rational fields.

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## Les Codes Quantiques (Survola)

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### Résumé

En 23 Octobre 2019, Google a annoncé avoir atteint la suprématie Quantique grâce au premier ordinateur Quantique Sycamore ayant 53 qubits. Cette réalisation est un fruit de plusieurs travaux pour transformer le langage informatique classique basé sur les bits au langage quantique basé sur les "Qubits". Ces efforts n'étaient jamais juste un jeu de loisir, mais plutôt une forte conviction que la machine classique atteint ses limites et que celles quantiques seront l'alternative pour répondre aux besoins de l'intelligence artificielle et aux ambitions des scientifiques en général.

En 1995, Calderbank et Shor [6], et puis Steane [9] en 1996 ont démontré la possibilité théorique de construire des bons codes quantiques. Mais la quantité des travaux sur ce sujet restait limitée car la machine quantique n'était qu'un rêve dont on était trop loin à réaliser.

Depuis 2014, les recherches en codes quantiques ont commencé à s'accélérer grâce surtout aux travaux de Ashraf et Mohammad [1] [2] [3], Gao et Wang [8], puis c'est Bag [4] [5] et Dinh [7] qui ont exploité le développement déjà réalisé en codage classique afin de le transformer en codage quantique en utilisant les techniques décrites par Calderbank [6]. Ce genre des Codes Quantiques est tout simplement les codes auto-duaux parmi les codes qui existent déjà.

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## The phase topology and bifurcation tori of the Generalized Yang-Mills potential

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### Abstract

In this paper, we study the Generalized Yang-Mills potential  $V = \frac{1}{2}(ax^2 + by^2) + \frac{1}{4}(cx^4 + dy^4) + \frac{1}{2}ex^2y^2$ . The structuring and evolution of the real phase space are explored. The bifurcation diagram is found and the bifurcations of solutions are discussed. The periodic solutions and their associated periods for singular common-level sets of the first integrals of motion are explicitly described. Numerical investigations are performed for the integrable case by means of Poincaré surfaces of section and comparing them with nearby living nonintegrable solutions, all generic bifurcations that change the structure of the phase space are illustrated, the problem can exhibit regularity-chaos transition over a range of control parameters of system.

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# ECC Fully homomorphic encryption scheme

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## Abstract :

In this paper we introduce a fully homomorphic encryption scheme on a ring  $R$ , in particular we are interested in cryptography based on conjugal classical problem in this ring.

We study the problem of conjugal over this non commutative ring.

The problem as stated is generally impossible to solve. Next, we describe a new encryption scheme over this ring based on this problem.

This method of encryption is based on two difficult problems; Discrete logarithm problem and conjugal classical problem.

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# Solving the closest vector problem with the reduced centered law method

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## Abstract :

In this work we expose a new method for the resolution of the closest vector problem in a Lattice. Given a vector  $U$  in  $\mathbb{R}^n$ , find a vector  $W$  of the Lattice ;  $\mathcal{L}$  which minimizes the distance between  $U$  and  $\mathcal{L}$ . This problem is NP hard.

Our contribution uses data analysis, making the vector  $W$  that we have to look for at a reduced centered vector  $V$ , the latter written in linear combination, whose integer coefficients are to be sought to give a rounded vector equal to the vector  $(1,1,\dots,1)$ .

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## Pólya fields of some real biquadratic number fields K

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### Résumé/Abstract

Let  $K$  be a number field and  $\mathcal{O}_K$  its ring of integers, denote by  $\prod_q(K)$  the product of all the prime ideals of  $\mathcal{O}_K$ , with norm  $q \geq 2$ .  $K$  is a Pólya field if  $\prod_q(K)$  are principal ideals of  $\mathcal{O}_K$ . In this work, we will determine the first cohomology group of units of  $K = \mathbb{Q}(\sqrt{d_1}, \sqrt{d_2})$ ,  $(d_1, d_2) = 1$  where  $N\epsilon_1 = N\epsilon_2 = -N\epsilon_3 = -1$ ,  $N\epsilon_i$  the norm of fundamental unit of  $k_i = \mathbb{Q}(\sqrt{d_i})$ , and we will conclude all Pólya fields of  $K$ .

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## Properties of twisted on the Ring $\mathbb{F}_q[e]$ , $e^2 = e$

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### Abstract

Let  $\mathbb{F}_q[e]$  be a finite field of  $q$  elements, where  $q$  is a power of a prime number  $p$ . In this paper, we study the Twisted Hessian curves denoted  $H_{a,d}(\mathbb{F}_q[e])$  over the ring  $\mathbb{F}_q[e]$ , where  $e^2 = e$  and  $(a, d) \in (\mathbb{F}_q[e])^2$ . Using the Twisted Hessian equation, we define the Twisted Hessian curves  $H_{a,d}(\mathbb{F}_q[e])$  and we will show that  $H_{\pi_0(a),\pi_0(d)}(\mathbb{F}_q)$  and  $H_{\pi_1(a),\pi_1(d)}(\mathbb{F}_q)$  are two Twisted Hessian curves over the field  $\mathbb{F}_q$ , where  $\pi_0$  and  $\pi_1$  are respectively the canonical projection and the sum projection of coordinates of  $X \in \mathbb{F}_q[e]$ . Precisely, we give a bijection between the sets  $H_{a,d}(\mathbb{F}_q[e])$  and  $H_{\pi_0(a),\pi_0(d)}(\mathbb{F}_q) \times H_{\pi_1(a),\pi_1(d)}(\mathbb{F}_q)$ .

### Keywords

Twisted Hessian curves, Elliptic Curves, Finite Ring, Finite field, Local ring, Cryptography.

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## A note on isodual constacyclic codes of length $p^s$ over the chain ring $\mathbb{F}_{p^m}[u]/\langle u^3 \rangle$

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### Abstract

Let  $\mathcal{R} = \mathbb{F}_{p^m}[u]/\langle u^3 \rangle$  be the finite commutative chain ring with unity, where  $p$  is a prime,  $m$  is a positive integer and  $\mathbb{F}_{p^m}$  is a finite field with  $p^m$  elements. In this paper, we determine all constacyclic codes of length  $p^s$  over  $\mathcal{R}$ , their sizes and their dual codes. As an application, we list some isodual constacyclic codes over  $\mathcal{R}$ .

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