Session : Algebra and Discrete Mathematics

Remarks On the Strong and Uniform Kreiss Resolvent Condition

A. AKRYM, A. ELBAKKALI and A. FAOUZI

LMF, Faculté des Sciences, Université Chouaib Doukkali, Eljadida.

Abstract

In the present paper, we extend the strong (uniform) Kreiss resolvent condition to a direct sum, and we show that if an operator on a Banach space satisfies the strong (uniform) Kreiss resolvent condition then so does the fractional powers of this operator.

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On AA-rings

A. Anebri[1] and N.Mahdou[2]

[1]Laboratory of Modelling and Mathematical Structures, University Sidi Mohamed Ben Abdellah, Fez
[2]Laboratory of Modelling and Mathematical Structures, University Sidi Mohamed Ben Abdellah, Fez

Abstract

In this paper, we introduce and study the class of rings in which every ideal consisting entirely of zero divisors is a d-ideal, considered as a generalization of strongly duo rings. Some results including the characterization of AA-rings are given in the first section. Further, we examine the stability of these rings in localization and study the possible transfer to direct product and trivial ring extension. In addition, we define the class of d_E-ideals which allows us to characterize von Neumann regular rings.

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Construction de l'$\infty-$champs géométrique des complexes parfaits

Brahim BENZEGHLI

Université Batna 2, Batna, Algérie

Résumé

Le but de ce travail est la construction d'une K-algèbre $\mathcal{A}$ par les morphismes

$$\begin{array}{llll}\gamma: & K[X_{.,.}^{\cdot}] & \rightarrow & K[X_{.,.}^{\cdot}]/I \\ & X_{.,.}^{l-1} & \mapsto & \gamma(X_{.,.}^{l-1}) \in \mathcal{M}_{r_l,r_{l-1}}(A)\end{array} \tag{1}$$

de sorte que $\gamma(X_{.,.}^{l}) \circ \gamma(X_{.,.}^{l-1}) = 0$.

Le schéma affine $V = V(I) := Spec(A)$ fournira un espace de modules pour les complexes parfaits strictes rigidifiés. Il y a un complexe universel strictement parfait et rigidifié de A-modules $\mathcal{C}_V$.

Soit B une K-algèbre, pour tout complexe strictement parfait rigidifié $\mathcal{C}$ des B-modules. On montre l'existence d'un unique morphisme d'anneaux $A \xrightarrow{\varphi} B$ et d'un unique isomorphisme de complexes $q : \mathcal{C} \rightarrow \mathcal{C}_{\mathcal{V}} \otimes_A B$. Autrement dit V paramétrise $\mathcal{C}$.

Le premier objectif est de montrer

— L'existence d'un unique morphisme $\mathcal{A} \xrightarrow{\varphi} \mathcal{B}$,

— L'existence d'un unique isomorphisme de complexes $q : \mathcal{C} \rightarrow \mathcal{C}_V \otimes_{\mathcal{A}} \mathcal{B}$ tel que

$$\forall i \in \{1, \cdots, n\} \quad q^i : \mathcal{B}^{r_i} \rightarrow \mathcal{C}_V \otimes_{\mathcal{A}} \mathcal{B} = \mathcal{A}^{r_i} \otimes_{\mathcal{A}} \mathcal{B} = \mathcal{B}^{r_i}$$

est l'identité.

Ce résultat montre que V paramétrise les C^i.

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Prime 3-uniform hypergraphs

Abderrahim Boussaïri[a 1] Brahim Chergui[a 2] Pierre Ille[b 3] Mohamed Zaidi[a 4]

[a]Faculté des Sciences Aïn Chock, Département de Mathématiques et Informatique,
Km 8 route d'El Jadida, BP 5366 Maarif, Casablanca, Maroc
[b]Aix Marseille Univ, CNRS, Centrale Marseille, I2M, Marseille, France

Abstract

Given a 3-uniform hypergraph H, a subset M of $V(H)$ is a module of H if for each $e \in E(H)$ such that $e \cap M \neq \emptyset$ and $e \setminus M \neq \emptyset$, there exists $m \in M$ such that $e \cap M = \{m\}$ and for every $n \in M$, we have $(e \setminus \{m\}) \cup \{n\} \in E(H)$. For example, $\emptyset$, $V(H)$ and $\{v\}$, where $v \in V(H)$, are modules of H, called trivial. A 3-uniform hypergraph is prime if all its modules are trivial. Given a prime 3-uniform hypergraph, we study its prime, 3-uniform and induced subhypergraphs. Our main result is : given a prime 3-uniform hypergraph H, with $v(H) \geq 4$, there exist $v, w \in V(H)$ such that $H - \{v, w\}$ is prime.

Key words : hypergraph, 3-uniform, module, prime.
AMS subject classification : (2010) : 05C65, 05C20.

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1. E-mail : aboussairi@hotmail.com
2. E-mail : cherguibrahim@gmail.com
3. E-mail : pierre.ille@univ-amu.fr
4. E-mail : zaidi.fsac@gmail.com

The Roots of Polynomial Sequence and Their Limiting Behaviour

Nouar Degaichi[1], H.Belbachir[2]

[1]Laboratoire CATI, Université Larbi tebessi, Tebessa
[2]Laboratoire CATI, USTHB , Alger

Résumé/Abstract

.The present work investigate the location of roots of polynomial familly that arise from Pascal triangle, it is defined by the recurrence formula :

$$\sum_{k=0}^{r} (-x)^k \binom{r}{k} T_{n-k} = y^r T_{n-r-q}$$

for all non zero real number x and y.
we focus our study to the familly defined by rows mod r
(i.e)

$$\sum_{k=0}^{r} (-x)^k \binom{r}{k} T_{n-k} = y^r T_{n-r}$$

we establish that for $r = 2$ the set of roots is empty and for $r = 3$ the set of roots of such familly satisfy the algebraic equation

$$\delta\lambda_1^n + \upsilon\lambda_2^n = \overline{\upsilon}\lambda_3^n$$

Where λ_1, λ_2 and λ_3 are the eigenvalues of the companien matrix.

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Characterization of automorphisms of $M = \bigoplus_{i=1}^{i=\infty} Ax_i$ having the extension property -where A is an integral domain and $M/Tor_A(M) \sim_A A^n$

Seddik ABDELALIM[1] , Abdelhak CHAICHAA[1] and Mostafa EL GARN[1]

[1] Laboratory of Topology, Algebra, Geometry and Discrete Mathematics, Faculty of Sciences Ain Chock, Hassan II University, Casablanca

Abstract

The extension property in the categorie of modules, is a very hard problem. However, there are some very important results in the group category, P.E.Schupp [6] proved that the automorphisms satisfying the extension property in the category of groups, characterize the inner automorphisms. Then, in order to generalize the result of Schupp, S.Abdelalim [1] characterized the automorphisms having the extension property, in the category of abelian groups. Then, what happens in categories other than abelian groups ? Can we have a similar result in the category of modules over an integral domain ? In this paper, we aim to extend the result in [1] to a special category of an infinite direct sum of cyclic modules. Let A be an integral domain. Let $n \in \mathbb{N}^*$ and M a direct sum of infinite cyclic modules over A such that $M/Tor_A(M) \sim_A A^n$. Consider α an automorphism of M. We will prove that α satisfies the extension property if and only if $\alpha = kid_M$, where k is a unit of A.

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A geometrical interpretation of the freeness of Heyting Algebras

A. Eladraoui[1], M. Kabil[2] and M. Pouzet[3]

[1] Laboratory of Algebra, Analysis and Applications Faculty of Sciences Ben M'sik University of Hassan II Casablanca.

[2] Laboratory of Mathematics and Applications,Faculty of Sciences and Technologies, Mohammedia, University Hassan II Casablanca.

[3] ICJ, Mathématiques, Université Claude-Bernard Lyon1, 43 bd. 11 Novembre 1918, 69622 Villeurbanne Cedex, France.

Abstract

In this work, we give a condition which ensures that a Heyting algebra minus the largest element is a free monoid. Our condition translates into a geometric property of injective envelopes of two-element metric spaces over the Heyting algebra. Our result generalizes the decomposition theorem established by Kabil et al.[8] in the setting of the Heyting algebra $F(A^*)$ consisting of final segments of the free monoid A^* equipped with the Higman ordering.

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THE TOTAL GRAPH OF AMALGAMATION

M. CHHITI, and K. KAIBA

Laboratory Modeling and mathematical structures, University S.M. Ben Abdellah, Fez

Résumé/Abstract

Let $f : A \to B$ be a homomorphism of commutative rings and let J be an ideal of B. The amalgamation of A with B along J with respect to f is the subring of $A \times B$ given by $A \bowtie^f J = \{(a, f(a) + j)| a \in A, j \in J\}$ This paper investigates the total graph of amalgamations. Our aim is to characterize when the graph is connected and compute its diameter and girth for various contexts of amalgamations. The new results yield new and original examples issued from amalgamations.

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Decomposability index of a Graph

A. Boussairi, I.Talbaoui

Laboratoire Topologie Algèbre Géométrie Mathématiques Discrètes, Université Hassan II, Casablanca

Abstract

An n-tournament T with vertex set V is simple if there is no subset M of V such that $2 \leq |M| \leq n-1$ and for every $x \in V \setminus M$, either $M \to x$ or $x \to M$. The arrow-simplicity of a tournament T is the minimal number $s(T)$ of arcs whose reversal yields a simple tournament. Müller and Pelant (1974) proved that $s(T) \leq \frac{n-1}{2}$, and that equality holds if and only if $n \equiv 3 \pmod 4$ and T is doubly regular. In this paper, a refinement of this bound is given for $n \not\equiv 3 \pmod 4$.

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On Monoid Algebras

[1]U.I.C. : Univesité Internationale de Casablanca, Bouskoura.
[2]C.R.M.E.F., Centre régionale des Métiers, de l'Education et de la Formation, Casablanca.

M. Zouagui[1] and L. Faouzi[2]

Abstract. In the theory of Boolean algebras, see for instance [K], a Boolean algebra B could be thought of as a Boolean lattice $< B, \wedge, \vee, 1, 0, - >$, where $< B, \wedge, \vee >$ is a distributive lattice with the neutral element 0 of $\vee$ and the neutral element 1 of $\wedge$, and each member x in B has its unique complement $\bar{x}$ such that x and its complement $\bar{x}$ satisfy $x \vee \bar{x} = 1$, $x \wedge \bar{x} = 0$ and $\bar{\bar{x}} = x$. The boolean algebra is also equipped with a partial order $\leq$ defined as $a \leq b$ if $a \wedge b = a$ for any elements a and b of B.
Boolean algebras could also be seen as k-algebras $< B, +, \cdot, 0, 1 >$ given by $x + y = x \wedge y - x \vee y$ (where $a - b = a \vee \bar{b}$ for any couple of elements a and b in B) and $x \cdot y = x \wedge y$ ($x \cdot y$ is often denoted by xy) (here, the field $k = \mathbb{Z}/2\mathbb{Z}$), see L. Heindorf [H1] and E. Evans [E].
The aim of this work is to extend all their results in the context of k-algebras, see for example [H1], [E] which helps to establish some properties of algebras over monoids (or semi group algebras) as algebras. Consequently, we derive some concrete representations.

Keywords : Monoid algebras, quasi-monoid algebras, Boolean algebras, Tail algebras, upper-lattice algebras, free algebras over posets, interval algebras.
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E-mail : mohamed.zouagui@uic.ac.ma (M. Zouagui).
E-mail : latifaouzi@hotmail.com (L. Faouzi).

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