Session : Analysis and optimal control

The output sensitivity problem for a discrete-time linear disturbed systems

Soukaina. Ben Rhila[1], Mustapha. Lhous[2], Mostafa. Rachik[3]

[1]Laboratory of Analysis Modelling and Simulation, Department of Mathematics and Computer Science, Faculty of Sciences Ben M'Sik, Hassan II University Casablanca, BP 7955, Sidi Othman, Casablanca, Morocco
[2]Laboratory of Modelling, Analysis, Control and Statistics, Department of Mathematics and Computer Science, Faculty of Sciences Ain Chock, Hassan II University of Casablanca, Morocco

Résumé /Abstract :

In this paper, we consider a linear perturbed system with discrete time. We introduce a new approach that studies output sensitivity where the initial state is infected by uncertain disturbances. The main goal of this paper is to determinate the set of possible gain matrices whose role is not only to make the system insensitive to all disturbances but to achieve a predefined stabilization mode. The characterization of this set is studied and an algorithm for determining the gain matrices is presented, some examples and numerical simulations being given to illustrate the result obtained.

Keywords: discrete-time, relatively insensitive, linear system, observability, stability, disturbance,

Références :

[1] Emilia Fridman. E_ects of small delays on stability of singularly perturbed systems. Automatica, 38(5):897_902, 2002.

[2] A Goubet-Bartholoméüs, M Dambrine, and JP Richard. Stability of perturbed systems with timevarying delays. Systems & Control Letters, 31(3) :155_163, 1997.

[3] Thierry Floquet, Jean-Pierre Barbot, and Wilfrid Perruquetti. Higher-order sliding mode stabilization for a class of nonholonomic perturbed systems. Automatica, 39(6) :1077_1083, 2003.

[4] Dan Chen and Dale E Seborg. Pi/pid controller design based on direct synthesis and disturbance rejection. Industrial & engineering chemistry research, 41(19) :4807_ 4822, 2002.

Analysis and optimal control of a multistrain SEIR epidemic model with saturated incidence rate and treatment

D. Bentaleb[1] and S. Amine[1]

[1] Laboratory of Mathematics and Applications, Faculty of Sciences and Technologies, University Hassan II of Casablanca, Po. Box 146, Mohammedia, Morocco.

Abstract

In this paper, we study the dynamic of a multi-strain SEIR model with both saturated incidence and treatment functions. Two basic reproduction numbers are extracted from the epidemic model, noted $R_{0,1}$ and $R_{0,2}$. Using the Lyapunov method, we investigate the global stability of the disease free equilibrium and prove that it is globally asymptotically stable when $R_{0,1}$ and $R_{0,2}$ are less than one. Moreover, we formulate the optimal control problem, solve it and perform some numerical simulations, to support the analytical results and test how well the proposed model may be applied in practice.

Keywords : SEIR ; multi-strain ; saturated incidence ; treatment function ; global stability ; optimal control.

Références

[1] Julien Arino, KL Cooke, P Van Den Driessche, and J Velasco-Hernández. An epidemiology model that includes a leaky vaccine with a general waning function. *Discrete and Continuous Dynamical Systems Series B*, 4(2) :479–495, 2004.

[2] Nicolas Bacaër. Daniel bernoulli, d'alembert and the inoculation of smallpox (1760). In *A short history of mathematical population dynamics*, pages 21–30. Springer, 2011.

[3] D. Bentaleb and S. Amine. Lyapunov function and global stability for a two-strain seir model with bilinear and non-monotone incidence. *International Journal of Biomathematics*, 2019.

[4] CP Bhunu. Mathematical analysis of a three-strain tuberculosis transmission model. *Applied Mathematical Modelling*, 35(9) :4647–4660, 2011.

[5] CP Bhunu and W Garira. A two strain tuberculosis transmission model with therapy and quarantine. *Mathematical Modelling and Analysis*, 14(3) :291–312, 2009.

[6] Jason M Brenchley, David A Price, Timothy W Schacker, Tedi E Asher, Guido Silvestri, Srinivas Rao, Zachary Kazzaz, Ethan Bornstein, Olivier Lambotte, Daniel Altmann, et al. Microbial translocation is a cause of systemic immune activation in chronic hiv infection. *Nature medicine*, 12(12) :1365, 2006.

[7] B Buonomo and D Lacitignola. On the backward bifurcation of a vaccination model with nonlinear incidence. *Nonlinear Analysis : Modelling and Control*, 16(1) :30–46, 2011.

[8] Vincenzo Capasso and Gabriella Serio. A generalization of the kermack-mckendrick deterministic epidemic model. *Mathematical Biosciences*, 42(1-2) :43–61, 1978.

[9] Anjana Das and M Pal. A mathematical study of an imprecise sir epidemic model with treatment control. *Journal of Applied Mathematics and Computing*, 56(1-2) :477–500, 2018.

[10] Zhilan Feng and Horst R Thieme. Recurrent outbreaks of childhood diseases revisited : the impact of isolation. *Mathematical biosciences*, 128(1-2) :93–130, 1995.

[11] Da-peng Gao and Nan-jing Huang. Optimal control analysis of a tuberculosis model. *Applied Mathematical Modelling*, 58 :47–64, 2018.

[12] JE Golub, S Bur, WA Cronin, S Gange, N Baruch, GW Comstock, and RE Chaisson. Delayed tuberculosis diagnosis and tuberculosis transmission. *The international journal of tuberculosis and lung disease*, 10(1) :24–30, 2006.

[13] Duane J Gubler. Epidemic dengue and dengue hemorrhagic fever : a global public health problem in the 21st century. In *Emerging infections 1*, pages 1–14. American Society of Microbiology, 1998.

[14] Herbert W Hethcote and P Van Den Driessche. Some epidemiological models with nonlinear incidence. *Journal of Mathematical Biology*, 29(3) :271–287, 1991.

[15] Zhixing Hu, Wanbiao Ma, and Shigui Ruan. Analysis of sir epidemic models with nonlinear incidence rate and treatment. *Mathematical biosciences*, 238(1) :12–20, 2012.

[16] Soovoojeet Jana, Swapan Kumar Nandi, and TK Kar. Complex dynamics of an sir epidemic model with saturated incidence rate and treatment. *Acta biotheoretica*, 64(1) :65–84, 2016.

[17] TK Kar and Soovoojeet Jana. Application of three controls optimally in a vector-borne disease–a mathematical study. *Communications in Nonlinear Science and Numerical Simulation*, 18(10) :2868–2884, 2013.

[18] TK Kar and Soovoojeet Jana. A theoretical study on mathematical modelling of an infectious disease with application of optimal control. *Biosystems*, 111(1) :37–50, 2013.

[19] William O Kermack and Anderson G McKendrick. A contribution to the mathematical theory of epidemics. In *Proceedings of the Royal Society of London A : mathematical, physical and engineering sciences*, volume 115, pages 700–721. The Royal Society, 1927.

[20] Muhammad Altaf Khan, Yasir Khan, and Saeed Islam. Complex dynamics of an seir epidemic model with saturated incidence rate and treatment. *Physica A : Statistical Mechanics and its Applications*, 493 :210–227, 2018.

[21] Suzanne Lenhart and John T Workman. Optimal control applied to biological models. *Chapman and Hall/CRC*, 2007.

[22] Wei-min Liu, Simon A Levin, and Yoh Iwasa. Influence of nonlinear incidence rates upon the behavior of sirs epidemiological models. *Journal of mathematical biology*, 23(2) :187–204, 1986.

[23] Dahlard L Lukes. *Differential equations : classical to controlled.* Elsevier, 1982.

[24] Oluwole Daniel Makinde. Adomian decomposition approach to a sir epidemic model with constant vaccination strategy. *applied Mathematics and Computation*, 184(2) :842–848, 2007.

[25] Kazeem O Okosun, Rachid Ouifki, and Nizar Marcus. Optimal control analysis of a malaria disease transmission model that includes treatment and vaccination with waning immunity. *Biosystems*, 106(2-3) :136–145, 2011.

[26] LS Pontryagin, VG Boltyanskii, RV Gamkrelidze, and EF Mishchenko. The mathematical theory of optimal processes, ser. translated from the russian by kn trirogoff ; edited by lw neustadt, 1962.

[27] Zhipeng Qiu and Zhilan Feng. Transmission dynamics of an influenza model with vaccination and antiviral treatment. *Bulletin of mathematical biology*, 72(1) :1–33, 2010.

[28] Helena Sofia Rodrigues, M Teresa T Monteiro, and Delfim FM Torres. Optimal control and numerical software : an overview. *arXiv preprint arXiv :1401.7279*, 2014.

[29] Shigui Ruan and Wendi Wang. Dynamical behavior of an epidemic model with a nonlinear incidence rate. *Journal of Differential Equations*, 188(1) :135–163, 2003.

[30] Nasser H Sweilam and Seham M AL-Mekhlafi. Numerical study for multi-strain tuberculosis (tb) model of variable-order fractional derivatives. *Journal of advanced research*, 7(2) :271–283, 2016.

[31] Douglas H Thomasey and Maia Martcheva. Serotype replacement of vertically transmitted diseases through perfect vaccination. *Journal of Biological Systems*, 16(02) :255–277, 2008.

[32] Pauline Van den Driessche and James Watmough. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical biosciences*, 180(1-2) :29–48, 2002.

[33] Jian-Jun Wang, Jin-Zhu Zhang, and Zhen Jin. Analysis of an sir model with bilinear incidence rate. *Nonlinear Analysis : Real World Applications*, 11(4) :2390–2402, 2010.

[34] Wendi Wang. Backward bifurcation of an epidemic model with treatment. *Mathematical biosciences*, 201(1-2) :58–71, 2006.

[35] Lih-Ing Wu and Zhilan Feng. Homoclinic bifurcation in an siqr model for childhood diseases. *Journal of Differential Equations*, 168(1) :150–167, 2000.

[36] Dongmei Xiao and Shigui Ruan. Global analysis of an epidemic model with nonmonotone incidence rate. *Mathematical biosciences*, 208(2) :419–429, 2007.

[37] Xu Zhang and Xianning Liu. Backward bifurcation of an epidemic model with saturated treatment function. *Journal of mathematical analysis and applications*, 348(1) :433–443, 2008.

A stochastic hepatitis B epidemic model driven by Lévy noise

Brahim Boukanjime[1] and Mohamed El Fatini[1]

[1]Ibn Tofail University, Faculty of Sciences, Department of mathematics, Kénitra, Morocco

Abstract

In this paper, we investigate a stochastic hepatitis B epidemic model driven by Lévy noise. We prove the existence of a unique global positive solution and we present sufficient conditions to derive extinction and persistence of the disease. Finally, numerical simulations illustrate the theoretical results.

Références

[1] B. Berrhazi, M. El Fatini,T. Caraballo R. Pettersson, *A stochastic SIRI epidemic model with Lévy noise*, DCDS-B. 23 (2018) 2415-2431.

[2] B. Berrhazi, M. El Fatini, A. Laaribi, R. Pettersson, *A stochastic SIRS epidemic model incorporating media coverage and driven by Lévy noise*, Chaos Solitons and Fractals. 105 (2017) 60-68.

[3] S.M. Ciupe, R.M. Ribeiro, P.W. Nelson, A.S. Perelson, Modeling the mechanisms of acute hepatitis B virus infection, J. Theoret. Biol., 247 (2007), pp. 23-35.

[4] K. Hattaf, N. Yousfi, A generalized HBV model with diffusion and two delays, Comp. & Math. with Appl. 69 (2015) 31-40.

Degenerate parabolic problems with variable exponent and L^1-data

A. SABRI[1], A. JAMEA[1,2] and H. ALAOUI[1]

[1]EMAPI, Faculté des Sciences, Université Chouaib Doukkali, Eljadida.
[2]entre Régional des Métiers de l'Education et de Formation Casablanca Settat.

Abstract

Let $\Omega \subset \mathbb{R}^d, (d \geq 2)$ be a open bounded domain with a connected Lipschitz boundary $\partial\Omega$ and T be a fixed positive real number. Our aim of this communication is to prove existence results of entropy solutions for the nonlinear degenerate parabolic problem with variable exponent

$$\begin{cases} \dfrac{\partial u}{\partial t} - \operatorname{div}\left(\omega|\nabla u|^{p(.)-2}\nabla u\right) = f \quad in \quad Q_T :=]0,\ T[\times\Omega, \\ \qquad u = 0 \quad on \quad \Sigma_T :=]0,\ T[\times\partial\Omega, \\ \qquad u(.,0) = u_0 \quad in \quad \Omega, \end{cases}$$

where $p(.)$ is a continuous function defined on $\overline{\Omega}$ with $p(x) > 1$ for all $x \in \overline{\Omega}$ and ω is a measurable function on Ω, strictly positive and satisfying the following hypotheses

(H_1) : $\omega \in L^1_{loc}(\Omega)$ and $\omega^{\frac{-1}{p(x)-1}} \in L^1_{loc}(\Omega)$,

(H_2) : $\omega^{-s(x)} \in L^1_{loc}(\Omega)$ where $s(x) \in \left(\frac{N}{p(x)}, \infty\right) \cap \left(\frac{1}{p(x)-1}, \infty\right)$.

The datum f is in $L^1(\Omega)$.

Références

[1] I. Aydin, *Weighted Variable Sobolev Spaces and Capacity,* J. Funct. Spaces Appl., **Vol. 17**,(2012), 17 pages.

[2] F. Andereu, J. M. Mazôn, S. Segura De leon, J. Teledo, *Quasi-linear elliptic and parabolic equations in L^1 with non-linear boundary conditions*, Advances in Mathematical Sciences and Applications **7** (1997), pp. 183–213.

[3] Ph. Bénilan, L. Boccardo, T. Gallouet, R. Gariepy, M. Pierre, J.L. Vazquez, *An L1 theory of existence and uniqueness of solutions of nonlinear elliptic equations*, Annali della Scuola Normale Superiore di Pisa **22** (1995), pp. 241–273.

[4] A. Jamea, A. Alaoui and A. El Hachimi : Existence of entropy solutions to nonlinear parabolic problems with variable exponent and L^1-data, Ric. Mat., Vol 67(2), 785-801 (2018).

[5] Y. H. Kim, L. Wang and C. Zhang, *Global bifurcation for a class of degenerate elliptic equations with variable exponents*, J. Math. Anal. Appl., **371** (2010), pp. 624–637.

[6] M. Sanchón and J.M. Urbano : Entropy solutions for the p(x)-Laplace equation. Trans. Amer. Math. Soc., 361 (2009) 6387-6405.

Etude paramétrique fréquentielle pour l'analyse vibratoire d'une plaque rectangulaire mince isotrope sous différentes conditions aux limites

I. Sama[1], A. El Ghoulbzouri[2]

Modélisation, Optimisation et Dynamique des Structures en Génie Civil MODSGC,
Université Abdelmalek Essaâdi , Al Hoceima

Résumé :

Les fréquences d'une plaque mince rectangulaire isotrope en vibration sont déterminées en utilisant la méthode de Rayleigh-Ritz et la solution du problème aux valeurs propres est obtenue en proposant une déformée de fonction forme de séries. Donc le système d'équations est résolu analytiquement, il est décrite mathématiquement par un système d'équations aux dérivées partielles avec différents conditions aux limites.

En outre, le modèle par la méthode des éléments finis de la plaque dans le système ABAQUS est développé, et des calculs numériques pour des plaques exemplaires sont réalisés. Les résultats des calculs de ces deux méthodes sont comparés.

Références :

[1] J. Guoyong, Y. Tiangui, S. Zhu, Structural vibration, Springer Berlin Heidelberg (2015) p37-58/p100-107.

[2] H. Takabatak, Simplified analytical methods of elastic plates, Springer Berlin Heidelberg (2018) p3-p23.

[3] T. Zarza, T. Benmansour, M. Naimi, Etude paramétrique fréquentielle pour l'analyse libre d'une plaque rectangulaire mince isotrope avec différentes combinaisons d'appuis en utilisant la méthode de Ritz, Sci. Technol. B Sci. Ing. (2007).

[4] A. Khennane, Introduction to finite element analysis using MATLAB and ABAQUS, CRC Press, Taylor & Francis Group (2013) p380-p419.

[5] K. Magnucki, D. Witkowski, E.Magnucka-Blandzi, Buckling and free vibrations of rectangular plates with symmetrically varying mechanical properties – analytical and FEM studies (2019).

[6] W. L. Li, X. Zhang, J. Du, Z. Liu, An exact series solution for the transverse vibration of rectangular plates with general elastic boundary supports. J. Sound Vib. 321, 254–269 (2009).