# Session : Applied Mathematics

# ESTIMATION NON-PARAMETRIQUE DE LA FONCTION DE RISQUE : CAS DES DONNEES FONCTIONNELLES SPATIALES

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**Dans cet article on propose d'estimer le point à haut risque dans $\mathcal{S}$ ( un compact $\mathcal{S} \in \mathbb{R}$ ), notée $\theta(x)$ défini par :**

$$\text{(I)} \qquad h^x\big(\theta(x)\big) = \max_{y \epsilon S} h^x(y)$$

**Où $h^x$ est la fonction de risque (voir [1])**

**Ce modèle a grand intérêt en statistique, notamment dans l'analyse de risque (voir [2]).**

**Dans notre contexte fonctionnel, on suppose qu'il existe un point unique $\theta(x)$ dans $S$** vérifiant (I).

L'estimateur naturel $\theta(x)$, noté $\widehat{\theta}(x)$, est tel que :

$$\text{(II)} \qquad \widehat{h}^x\left(\widehat{\theta}(x)\right) = \max_{y \epsilon S} \widehat{h}^x(y)$$

**Notre objectif d'estimer le point à haut risque $\theta(x)$ par la méthode locale linéaire pour des données spatialement dépendantes. Nous établissons la convergence presque complète de l'estimateur construit : , sous des conditions générales de régularité nous dérivons que notre estimateur possède de bonnes propriétés asymptotiques.**

**Mots clés**: **Spatial functional data, Local linear estimation, Point at high risk, Conditional hazard function, strongly mixing process.**

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# A GENERALIZATION OF D'ALEMBERT'S OTHER FUNCTIONAL EQUATION ON SEMIGROUPS

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## Résumé / Abstract :

Given a semigroup S generated by its squares and equipped with an involutive automorphism σ and a multiplicative function μ: S\to C such that μ(xσ(x)) = 1 for all x\inS, we determine the complex-valued solutions of the following functional equation

$$f(xy)-\mu(y)f(\sigma(y)x) = g(x)h(y), \quad x, y\in S.$$

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# A mathematical model of Moroccan Aristeus Antennatus fishery

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**Résumé/Abstract**

This paper describes a model of the interaction between the Aristeus Antennatus and Sardine marine species in two different areas : the first one is a preserved area against fishing and the second one is a free access fishing area. The Aristeus Antennatus in the preserved area grows according to the logistic model. If the Aristeus Antennatus population is in the preserved zone then it is protected against fishing but if not, i.e, if it is in the free acces fishing zone, it is captured. This paper has as objective to study the existence and to prove the equilibrium points stability by using eigenvalues analysis. As results, we found that the conditions that ensure the existence of the Aristeus Antennatus and Sardine marine populations are hold, and their coexistence is shown in the numerical simulations results.

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# Vers une approche d'algèbre linéaire pour des requêtes analytiques et efficaces des graphes RDF

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## Résumé:

RDF(Resource Description Framework) est un modèle de graphe pour représenter les informations sur le Web, tandis que SPARQL(Protocol and RDF Query Language) est un langage de requête pour manipuler les données RDF, mais ce dernier engendre une problématique sur la difficulté d'exécution des requêtes complexes lorsqu'il y'a un gros volume des données RDF, pour régler cette problématique l'approche algèbre linéaire[1-2] s'intervient, pour améliorer l'interrogation des données RDF, d'une manière optimisée avec un temps de réponse minimal. l'objet de cet article est de comparer , d'analyser et d'évaluer la capacité des travaux existants qui suit cette approche est-ce qu'ils sont pertinents dans le présent.

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# Energy harvesting in an excited van der Pol device using a delayed piezoelectric coupling mechanism

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## Abstract:

We investigate quasi-periodic (QP) vibration-based energy harvesting (EH) in a nonlinear device consisting of an excited van der Pol oscillator coupled to a delayed piezoelectric coupling mechanism. The governing equation for the harvester system can be written in the dimensionless form as

$$\ddot{x}(t)+\dot{x}(t)-[\alpha-\beta x(t)^2]\dot{x}(t)-\chi v(t)=f\cos(\omega t) \quad (1)$$

$$\dot{v}(t)+\lambda[v(t)-v(t-\tau)]+\kappa\dot{x}(t)=0 \quad (2)$$

where x(t) is the relative displacement of the rigid mass m, f, ω are, respectively, the amplitude and the frequency of the harmonic excitation, v(t) is the voltage across the load resistance, α and β are the mechanical damping ratio, χ is the piezoelectric coupling term in the mechanical attachment, κ is the piezoelectric coupling term in the electrical circuit, λ is the reciprocal of the time constant of the electrical circuit and τ is the time delay in the electric circuit. Here we explore the effect of a time delayed introduced in the piezoelectric subsystem on the EH performance of the system. We consider the case of primary resonance for which the frequency of the harmonic excitation is near the natural frequency of the oscillator. Analytical approximation of the QP response and the corresponding power output are obtained using the double-step multiple scales method. The effect of time delay on the EH performance is studied; It is shown that for appropriate combination of time delay parameters, QP vibration can be used to scavenge energy over a broadband of the excitation frequency away from the resonance with a significant performance. An optimum range of the system parameters where the QP vibration-based EH is maximum is determined. Numerical simulations are conducted to support the analytical predictions.

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# Discrete Laplace operator and its essential self-adjointness on a weighted 3-simplicial complex

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## Résumé /Abstract :

We define the structrue of a simplicial complex of dimension 3 (3-simplicial complex) and also its weighted version (weighted 3-simplicial complex). After, we define the discrete operators between the sets of cochains of dimension 0,1,2 and 3. Moreover, we give the Gauss-Bonnet operator of a weighted 3-simplicial complex. In addition, we introduce the discrete Laplace operator of our weighted 3-simplicial complex. furthermore, we use the cut-off functions in order to establish a geometric hypothesis for the $\chi$-completeness of a weighted 3-simplicial complex. Finally, we prove that our discrete Laplace operator of a weighted 3-simplicial complex is essential self-adjoint.

**Key words :** Weighted 3-simplicial complex, discrete operators on cochains, Gauss-Bonnet operator, discrete Laplace operator of a weighted 3-simplicial complex, $\chi$-completeness of a weighted 3-simplicial complex, essential self-adjointness.

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# Improvements of some iterative algorithms for a common solution of split equilibrium problems, variational inequality and fixed points problems.

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**Abstract**

Throughout this paper, we suggest and study a new iterative method for solving a split equilibrium problem and fixed point problem of a finite family of a nonexpansive mapping in a real Hilbert space. Then we prove the strong convergence of our algorithm under some suitable assumptions, to the common element of the solution set of split equilibrium problems and the set of fixed points problems in the setting of Hilbert spaces. Furthermore, we give some numerical experiments to illustrate the efficiency of our proposed iterative method. Due to its numerous applications such as dynamical games, color imaging and intensity modulated radiotherapy, host of researches had been developed to solve this class of problems, therefore, all the results presented in this paper improve some existing methods in the earlier and recent literature.

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# Fixed point Theorems for Generalized $\theta-$ contraction and $F-$contraction on $S-$metric spaces.

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**Abstract**

The fixed point theory was started with the classical Banach contraction principle . This principle has been generalized using different approaches.
In 2012, Sedghi et al. [1] introduced the concept of $S-$metric space by modifying $D-$metric and $G-$metric spaces and proved some fixed point theorems for a self-mapping on a complete $S-$metric space.
in this paper we prove a fixed point theorem and a using the $\theta-$ contraction and $F-$contraction on $S-$metric spaces.

**Definition 1.** *Let $X$ be a nonempty set. An $S-$metric on $X$ is a continuous function $S: X^3 \to R^+$ such that satisfies the following conditions for each $x, y, z$,and $a \in X$:*
$(S_1)$ $S(x,y,z) > 0$ *for all* $x, y, z, \in X$ *with* $x \neq y \neq z$ *or* $x \neq y$ *or* $x \neq z$ *or* $y \neq z$,
$(S_2)$ $S(x,y,z) = 0$ *if and only if* $x = y = z$,
$(S_3)$ $S(x,y,z) \leq S(x,x,a) + S(y,y,a) + S(z,z,a)$,
$(S_4)$ $S(x,y,z) = S(x,z,y)$, $S(x,y,z) = S(y,x,z)$ *and* $S(x,y,z) = S(z,y,x)$. *(symmetric)*

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# Non-continuous double barrier reflected BSDEs with jumps and stochastic Lipschitz coefficient

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**Abstract**

We consider a doubly reflected backward stochastic differential equations with jumps where the lower barrier and the opposite of the upper barrier are assumed to be right upper-semicontinuous (not necessarily càdlàg). We prove the existence and uniqueness of the solution when the coefficient is stochastic Lipschitz by using an equivalent transformation which is a coupled system of one-reflected backward stochastic differential equations. We give also a comparison theorem for the solutions.

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# Étude théorique et approximation numérique d'un problème à frontière libre de type Bernoulli intérieur

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**Résumé**

Ce travail a porté sur l'étude d'un problème à frontière libre de type Bernoulli intérieur qui est apparu dans la modélisation de plusieurs phénomènes. Nous citons par exemple le problème qui consiste à construire une couche isolante minimisant les fuites de chaleur ou de courant soumises à une quantité donnée de matériau isolant. Ou bien le problème de détection de la forme d'une inclusion située à l'intérieur d'un matériau ou d'un système.
Pour l'étude théorique et numérique, ce problème est reformulé en un problème d'optimisation de forme géométrique. Deux types de fonctionnelles coûts sont considérés. La première de type classique de moindres carrés et la deuxième de type Kohn-Vogelius.
Nous avons commencé par l'étude de l'existence et l'unicité de forme optimale. Ensuite, nous avons montré un résultat de différentiabilité de la solution d'état. Puis, nous avons calculé la dérivée de forme de la solution du problème d'état. Ainsi, nous avons calculé les gradients des deux fonctionnelles coûts en introduisant un problème adjoint pour la méthode de moindres carrés. Par ailleurs, nous avons montré que le gradient de la fonctionnelle coût de Kohn-Vogelius dans une direction donnée dépend uniquement de la solution d'état et non pas de ses dérivées.
Concernant l'approximation du problème d'optimisation de forme, nous avons utilisé la méthode des éléments finis pour la discrétisation du problème direct et la méthode du gradient pour la minimisation. Finalement des résultats numériques sont représentés en utilisant la méthode géométrique, et nous avons comparé les résultats obtenus par les deux fonctionnelles de formes.

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