Session : Harmonic Analysis

An analog of V. A. Abilov's theorem in generalized Bessel harmonic analysis

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Résumé/Abstract

En 2008, V. A. Abilov, F. V. abilova et M. K. Kerimov montrèrent via l'opérateur de Steklov, deux utiles estimations de la transformée de Fourier d'une classe de fonctions de l'espace $L_2(\mathbb{R})$.

En 2009, les mêmes auteurs, moyennant l'opérateur de translation associé à l'analyse harmonique de Bessel, prouvèrent des estimations analogues à la transformée de Fourier-Bessel d'une classe de fonctions de l'espace $L_2(\mathbb{R}^+, x^{2\alpha+1}dx)$.

En 2013, les Professeurs M. El Hamma et R. Daher étendirent les estimations de la transformée de Bessel à une classe de fonctions de l'espace $L_p(\mathbb{R}^+, x^{2\alpha+1}dx)$. avec $1 < p \leq 2$.

Dans l'optique de ce travail, nous y montrons des résultats analogues, en donnant des estimations de la transformée de Fourier-Bessel généralisée d'une classe de fonctions appartenant à l'espace $L_p(\mathbb{R}^+, x^{2\alpha+1}dx)$ avec $1 < p \leq 2$.

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Quaternion-octonion algebra and harmonic analysis

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Résumé/Abstract

There have been several efforts in the literature to extend the traditional Fourier transformation by using the quaternion or octonion algebra. The aim of this talk is to present the quaternion Fourier and the octonion Fourier analysis. We derive some properties of these transforms.

On Titchmarsh's theorem

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Résumé/Abstract

Using a generalized dual translation operator, we obtain an analog of Titchmarsh' s theorem for the generalized Dunkl transform for functions satisfying the Q-Dunkl Lipschitz condition in the space L^2_Q

Inequalities of Hardy-Littelwood for the Hausdorff's operator

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Résumé/Abstract

The search for operators bounded on spaces of functions, and more precisely those using the classical Fourier transform on spaces Lp, has constituted since Hardy's work, one of the main concerns of harmonic analysis. This brief presentation illustrates the example of the Hausdorff operator.

Jacobi transform of (ν, γ, p)-Jacobi Lipschitz functions in the Space $L^p(\mathbb{R}^+, \Delta_{(\alpha,\beta)}(t)dt)$

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Dedicated to Professor Radouan Daher for his 61's birthday

Abstract : Using a generalized translation operator, we obtain an analog of Younis Theorem 5.2 in [**?**] for the Jacobi transform transform for functions satisfying the (ν, γ, p)-Jacobi Lipschitz class in the space $L^p(\mathbb{R}^+, \Delta_{(\alpha,\beta)}(t)dt)$.

Keywords : Jacobi operator, Jacobi transform, generalized translation operator.

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Extension of the Theorem of Brun-Titchmarsh

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Résumé/Abstract

In this talk, we will present a result, collaborated with Professor Abdelaziz RAOUJ, concerning an upper bound of the sum

$$\sum_{\substack{x-y<n\leq x \\ n\equiv a\,(mod\ k) \\ \omega(n)\leq s}} f(n),$$

uniformly for $x^{\alpha} < y \leq x$ such that : $0 < \alpha < \frac{1}{2}$, s is an integer greater than 1 ; a, $k \in \mathbb{N}$ with $(a,k) = 1$ and $\omega(n)$ denote the number of the different prime factors of n and f is a non-negative multiplicative function satisfying the following conditios :

(i) There exists a positive A_1 such that

$$f(p^l) \leq A_1^l, \qquad \text{for all primes}\, p \text{ and } l \geq 1.$$

(ii) For any $\epsilon > 0$, there exists a positive constant A_2, such that

$$f(n) \leq A_2 n^{\epsilon}, \qquad n \geq 1.$$

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Dunkl-Hausdorff operators on $BMO_\alpha(\mathbb{R})$ and $W_\alpha^{p,r}(\mathbb{R})$

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Résumé/Abstract

In this talk, we study the Dunkl-Hausdorff operators $\mathcal{H}_\alpha$ on the Dunkl-type spaces of functions of bounded mean oscillation $BMO_\alpha(\mathbb{R})$ and Dunkl-type Sobolev spaces $W_\alpha^{p,r}(\mathbb{R})$. we give simple sufficient conditions for these operators to be bounded on these spaces.

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Sur Les variétés Lorentziennes Admettant Un Champ de Droites Parallèle Dégénéré

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Résumé /Abstract :

Les solitons de Ricci lorentziens ont fait l'objet d'études approfondies, montrant de nombreuses différences essentielles par rapport au cas riemannien. En fait, bien qu'il existe des solitons de Ricci homogènes riemanniens tridimensionnels, il n'y a pas de solitons de Ricci riemanniens invariants à gauche sur les groupes de Lie de dimension trois. De plus, le cas Lorentzien est beaucoup plus riche, permettant l'existence de « expanding, steady and shrinking » solitons de Ricci invariants à gauche. Ces résultats rendent intéressant d'étudier plus avant le soliton de Ricci sur les variétés Lorentziennes. Le but de cet exposé est de prouver l'existence de solitons de Ricci non triviaux (c'est-à-dire pas d'Einstein) sur des variétés de Lorentzian Walker quadridimensionnelles qui ne sont pas conformément plates, en outre, nous montrons que seuls les solitons de Ricci « steady » peuvent être des gradients solitons de Ricci.

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An analog of Titchmarsh's theorem for the q-Dunkl transform in the space $L^2_{q,\alpha}(\mathbb{R}_q)$

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Abstract :

In [1], by the use of the q^2-analogue differential operator studied in [2], Bettaibi et al. introduced a new q-analogue of the classical Dunkl operator and studied its related Fourier transform, which is a q-analogue of the classical Bessel-Dunkl one and called the q-Dunkl transform.

Using the q-harmonic analysis associated with the q-Dunkl operator, we prove an analog of Titchmarsh's theorem for functions satisfying the q-Dunkl Lipschitz condition, using a generalized q-translation operator in the space $L^2_{q,\alpha}(\mathbb{R}_q)$.

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